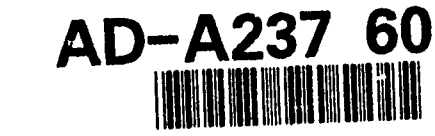

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

THE EXPLORATION OF AN ALTERNATIVE TO ACCEPTANCE SAMPLING

by

Craig A. Hammons

June 1990

Thesis Advisor G. F. Lindsay

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The Exploration of an Alternative
to Acceptance Sampling

by

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Lieutenant, United States Navy
B.S., Oklahoma State University, 1984

Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

This thesis considers the problem of evaluating a producer's program in statistical process control, from the standpoint of the consumer. A model is postulated reflecting the variability in proportion nonconforming of a process and the characteristics of the final control chart in the process. From this the steady-state solution to a Markov chain is used to find the output proportion nonconforming of the process.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

The commander of the Naval Weapons Support Center in Crane, Indiana is concerned with the quality level of projectiles stored at this base. A projected manpower shortage there may require the use of alternative methods to assure the quality of these projectiles with fewer personnel. The base commander also questions the capability of the current methods being employed to provide him with accurate estimates of quality.

For many years, acceptance sampling has been employed as a primary tool to provide assurance that high quality of goods is accepted by the military. Some military standards provide the sampling procedures, sample size, and decision rules by which a given shipment of goods, usually called a lot, is either accepted or rejected. These standards typically require that the sample or samples be drawn randomly from the lot of goods. These methods have several weaknesses. One drawback is that an entire lot must have been produced before a decision can be made to accept or reject it. If a lot is rejected, delays may occur before acceptable goods can be acquired. In some cases the producer's own quality programs are so effective that acceptance sampling may be unnecessary due to such a high rate of acceptance. One alternative to acceptance sampling is to conduct a program that reviews and evaluates the quality control procedures already in place at the manufacturer's facility. If the quality personnel at the Navy base approve the procedures used by the manufacturer, then they may choose to accept the goods from the manufacturer's operation without further inspection. In this thesis we examine one method for evaluating a quality control procedure. Approval of a producer's quality control procedures could be as effective as acceptance sampling in providing quality to the Navy and might reduce the number of quality assurance personnel needed at the Naval Weapons Support Center.

In Chapter II we examine the current quality methods that are being used at the base for acceptance sampling.

In Chapter III, we discuss the differences between statistical quality control and statistical process control. This leads us to the exploration of using control charts as a means of helping to control the level of quality in a process. To examine the effect of a control chart on a typical manufacturing process, we first create a stochastic model of a manufacturing process. The model employs a Markov chain to describe how the behavior of the process is affected when a control chart procedure is imposed on it. The

decision rules of the control chart are embedded in the model. A small computer program encodes the model. Given the necessary inputs, one can use this program to compute the output quality level of the manufacturing process. The results of the program are included in this thesis. They provide a rough measure of how the process and control chart parameters might impact the quality of the output of the manufacturing process.

In Chapter IV, cost and effectiveness measures are used to determine whether or not the proposed method of statistical process control could be more desirable than the current method of statistical quality control. The cost comparison is essentially a qualitative argument. The effectiveness comparison is quantitative.

In Chapter V the results of the study are summarized and recommendations are made pertaining to the use of alternative quality assurance methods at the Naval Weapons Support Center.

II. ACCEPTANCE SAMPLING: DESCRIPTION AND ANALYSIS

In this chapter, the importance of an effective quality assurance program is addressed. Also, current methods used in acceptance sampling are explored with the primary focus being on MIL-STD 105D. MIL-STD 414, another set of acceptance sampling plans, is also discussed and comparisons are made between MIL-STD 105D and MIL-STD 414.

Acceptance sampling may be used by a consumer in deciding to accept or reject a shipment of goods from a manufacturer. If the shipment of goods is rejected, it will often be sent back to the manufacturer. If a manufacturer's product is being rejected at a high rate, one of two things may happen. The manufacturer may take steps to improve his production methods, or the customer may find a better source of supply. Besides helping to ensure quality in accepted goods, acceptance sampling indirectly improves quality of production through its encouragement of good quality by requiring a high rate of acceptance.

A. MIL-STD 105D SAMPLING PLANS

Some of the most popular acceptance sampling plans are those in MIL-STD 105D. MIL-STD 105D was developed in 1950. Many of the sampling tables and procedures used in MIL-STD 105 were derived from the Army Service Forces Tables developed in 1942 by Bell Telephone Laboratories for the Ordnance Department of the United States Army. Another source for the development of MIL-STD 105 was the JAN (Joint Army and Navy) Standard 105 Statistical Sampling Tables and Procedures originally developed for the Navy by the Statistical Research Group of Columbia University in 1945. MIL-STD 105D, the fourth revision to the standard, was adopted in 1963 by the ABC Working Group, a committee made up of members from the military agencies of the United States, Great Britain, and Canada. This was the last revision to the standard. [Ref. 1]

MIL-STD 105D is essentially a collection of sampling plans or a sampling scheme. The plans in this standard only use attributes data. The focal point of the standard is the acceptable quality level or AQL. The AQL is "the maximum percent defective (or the maximum number of defects per hundred units) that, for purposes of sampling inspection, can be considered satisfactory as a process average " [Ref. 2]. In applying the standard, it is expected that there will exist a clear understanding between the producer

and the customer as to what the customer considers the acceptable quality level for a given product. Although an obvious purpose of an acceptance sampling plan is to ensure that the customer receives a product of at least acceptable quality, an effect of using the plan "is in general to force the supplier to submit product of such a quality that a small percentage of the lots submitted for inspection are rejected" [Ref. 1].

Another characteristic of the standard is inspection level. The inspection level determines the relationship between lot size and sample size. Three general levels of inspection are offered. Under ordinary circumstances, Level II is used. In instances, where the quality of goods is high, we may specify Level I, which will provide less discrimination via a smaller sample size. When the quality of goods is low, we may specify Level III, which will provide more discrimination via a larger sample size. There are also four special inspection levels offered: S1, S2, S3, and S4, which offer smaller sample sizes than Level I. These special levels may be used where relatively small sample sizes are necessary and large sampling risks can or must be tolerated. The inspection level is adopted at the initiation of the sampling program and is generally not changed thereafter. [Ref. 2]

MIL-STD 105D also offers three different types of sampling plans to choose from: single sampling, double sampling, and multiple sampling plans. The choice of a plan is frequently based on cost comparisons between the difficulties in conducting the plans and the average sample sizes of the plans. The average sample size for single plans is generally greater than that of either double or multiple plans. However, the administrative difficulty and average cost per unit of the sample are usually less with the single plan than with either the double or multiple plans. [Ref. 1] We only consider single sampling plans in this thesis.

For a specified AQL, inspection level, and lot size, MIL- STD 105D gives a normal sampling plan that is to be used as long as the supplier is producing a product of AQL quality or better. It also gives a tightened plan to shift to if there is evidence that a deterioration in quality has occurred. The rule is that a switch from the normal plan to the tightened plan will be made if two out of five consecutive lots have been rejected on original inspection. Normal inspection is re-instituted if five consecutive lots have been accepted on original inspection. If ten consecutive lots remain under a tightened plan, inspection is stopped pending action on quality. [Ref. 1] Similarily, MIL-STD 105D calls for shifts to reduced inspection if the quality is observed to be especially good. The rule here is that production must have been running at a steady rate and the last 10 lots must have been accepted on original normal inspection. In order to shift to reduced

inspection, the cumulative number of defective items in the last 10 samples must be less than the value set forth in the MIL-STD 105D table pertaining to limit numbers for reduced inspection. [Ref. 1] The basic idea of reduced inspection is to save the consumer money when quality has been consistently high.

The following is an example of one sampling plan from MIL-STD 105D. Suppose lots of size 1000 have been specified, an AQL of 1.5 percent has been agreed to, and a discrimination Level II is desired. For lot sizes of 1000, the single sampling plans set forth in MIL-STD 105D require a sample of size 80. The lot is accepted if there are 3 or fewer defective items in the sample and rejected if there are 4 or more defective items [Ref. 3]. Each sampling plan in MIL-STD 105D generates an operating characteristic (OC) curve. An OC curve gives the probability of accepting a lot of a given quality. Just as with any OC curve, there are Type I and Type II errors involved. In our case, the probability of a Type I error is the probability of rejecting a lot as a function of the quality of the lot (proportion non-conforming). The probability of a Type II error is the probability of accepting a bad lot, where "bad" corresponds to a larger proportion nonconforming that the specified AQL. The OC curve for our example is shown in Figure 1.

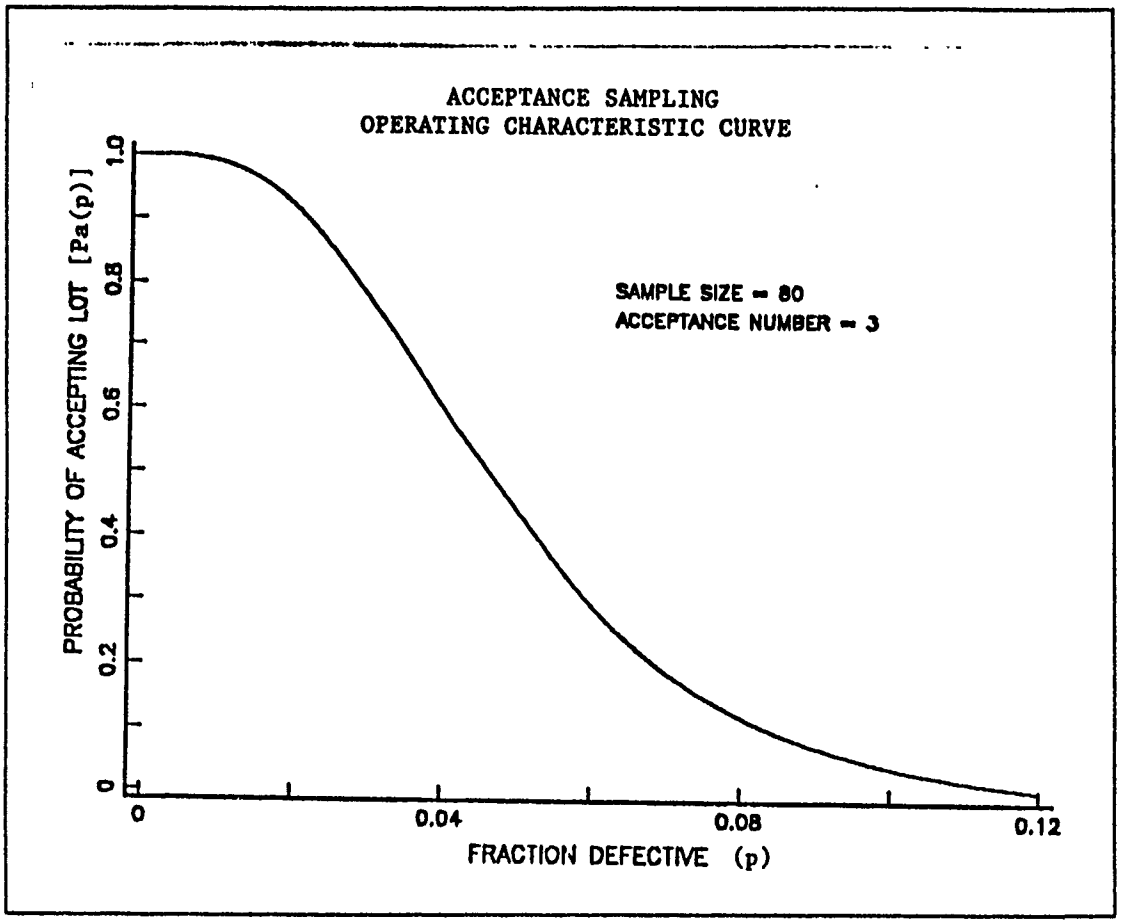

Figure 1. Acceptance Sampling OC Curve

For this OC curve, α , or the probability of rejecting a lot of good quality (AQL = .015) is equal to approximately .01. Similarly, β , or the probability of accepting a bad lot (p = .06, for example) is equal to approximately .25.

B. MIL-STD 414 SAMPLING PLANS

Another frequently used set of sampling plans is found in MIL-STD 414. All of the sampling plans in this standard use variables data, and it is assumed that the variables have a normal distribution. It is one of several variables sampling plans. This standard is not used at the Naval Weapons Support Center. MIL-STD 414 is an example of a variables sampling plan for situations where the quality requirement is still specified as

specified as a percent nonconforming. [Ref. 4]

Some points of similarity between MIL-STD 105D and MIL-STD 414 are:

- both are based on the concept of AQL,
- both pertain to lot-by-lot acceptance inspection,
- both provide procedures for normal, tightened, and reduced inspection,
- both provide several inspection levels,
- in both sets of plans, the sample size is greatly influenced by lot size, and
- in both sets of plans, all plans are identified by a sample size code letter.

MIL-STD 414 is also similar to MIL-STD 105D in that in both sets of plans sampling plans under normal inspection were designed to protect the producer by rejecting with small probability those lots which have a percent defective no larger than the specified AQL. Just as in MIL-STD 105D, the protection to the customer depends largely on the use of tightened inspection whenever the process average appears to be unsatisfactory or there are other reasons to be suspicious of the process. [Ref. 3]

The primary advantage of using MIL-STD 414 is that the same OC curve can be obtained with a smaller sample size than is required by Standard 105D. The precise measurements required by a plan from Standard 414 will probably cost more than the simple classification of items required by a plan from MIL-STD 105D, but the reduction in sample size will hopefully offset this extra expense. For example, a plan from MIL-STD 105D may require a sample of 50, but a comparable plan from Standard 414 requires a sample of only 30. If the unit cost of measurement is less than 5 3 times that of simply classifying the items, the MIL-STD 414 sampling plan will effect a savings. [Ref. 3] Such savings may be especially significant if inspection is destructive and the item is expensive.

The primary disadvantage of a sampling plan from MIL-STD 414 is that a separate plan must be employed for each quality characteristic that is being inspected. For example, if an item is to be inspected for five quality characteristics, then it would be necessary to have five separate inspection plans from MIL-STD 414; whereas, acceptance or rejection of the lot as a whole could be based on a single sampling plan from Standard 105D. Also, it is theoretically possible, although unlikely, that under a variables plan a lot will be rejected by the variables criteria, even though the sample actually contains no defective items. Another disadvantage of a variables sampling plan is that the quality characteristic must come from a normal distribution. Since the quality characteristics

being sampled will not always be normally distributed, the use of MIL-STD 414 will not always be appropriate.

Due to the higher degree of popularity and use of MIL-STD 105D as opposed to MIL-STD 414, plus the fact that Standard 105D is used at the Naval Weapons Support Center, MIL-STD 105D will be used as the benchmark with which to compare any proposed methods in later chapters.

C. MEASURING THE PERFORMANCE OF A SAMPLING PLAN

The most basic measure of a sampling plan's performance in monitoring quality is its OC curve, which shows the probability that a lot of given quality will be accepted. It is also useful to have a measure of the quality of the product that will reach the consumer's shelves after acceptance sampling has taken place.

We will assume that rectifying inspection is employe n the sampling process. What is meant by this is that all rejected lots will be inspect ercent and nonconforming items will be replaced by good ones. Thus, with th' mption, as lots of size L pass through the sampling process, one of the following scenarios will occur:

1. The lot is accepted, or
2. The lot is initially rejected; but, is subjected to 100 percent inspection and bad items are replaced with good ones. The lot is then accepted.

The sample s will eventually contain no defective items since all defects in the sample are replaced by good ones. The same is true for the lot if it is initially rejected. It will then be inspected 100 percent with the bad items being replaced by good ones. If the lot is initially accepted, it will contain $p(L-s)$ defective items, where p is the proportion of defective items in the lot. This will occur Pa proportion of the time. Thus, the sampling plan will on the average, turn out lots that contain $Pa(p(L-s))$ defective items (Pa is the probability of accepting a lot and is taken from the OC curve for the sampling plan). If we divide by the lot size, L we can express this average outgoing quality as a fraction defective as follows:

$$AOQ = \frac{Pa(p(L-s))}{L} \quad \text{[Ref. 1].}$$

Thus, this concept of average outgoing quality, or AOQ, gives us a method of examining the effect that a particular acceptance sampling plan has on product quality. AOQ is a function of p. It has a maximum value which is called the average outgoing quality limit, AOQL. Tables are provided in MIL-STD 105D that give the factors with which

the AOQL values for specific sampling plans can be calculated. The AOQL is a value representing the worst case AOQ for a particular sampling plan. The value of p corresponding to the AOQL is of particular importance. This subject will be addressed further in Chapter IV.

A producer may depend on either his or a consumer's quality control department for post-production inspection of his products. This inspection results in nonconforming items being screened out and replaced by conforming items. Often, this is how sampling plans such as MIL-STD 105D are used in a manufacturing operation. However, this *post*-production type of inspection is wasteful because it allows time and materials to be invested in products which are not always saleable.

One way to solve this problem, is to adopt an inspection strategy wherein a product can be inspected for quality in the early and middle stages of production. A second way of solving this problem is the adoption of a strategy of prevention. Clearly, it is much more effective to avoid waste by not producing unsaleable output to begin with. What follows in the next chapter is the basis for such a strategy.

III. A DIFFERENT APPROACH - DESCRIPTION AND ANALYSIS

In this chapter, the differences between statistical quality control and statistical process control are addressed. These differences lead to another outlook towards quality assurance and an alternate method of dealing with it which is set forth in this chapter. The fundamental idea of the alternate method is the use of control charts by the producer to gain *statistical control* of the manufacturing process with the result ultimately being better quality in the product. Since a control chart is a form of control based upon sampling rather than 100 percent inspection, the quality of process output will depend not only on the control chart, but upon the characteristics of the process which cause its likelihood to produce defects to vary over time. Thus, a model will be proposed to estimate the performance of the manufacturing process and a Markov chain will be used to develop a measure of process output. Both of these will be presented in detail.

A somewhat similar study was conducted by Brugger and is detailed in his technical paper [Ref. 5]. Brugger did use a Markov chain approach; however, his study was focused on modeling an acceptance sampling procedure vice a control chart procedure.

A. STATISTICAL QUALITY CONTROL VERSUS STATISTICAL PROCESS CONTROL

Quality assurance will now be approached in a different manner than in acceptance sampling. It will be addressed from the manufacturer's point of view. Aside from rectifying inspection, quality cannot be inspected into an item after it has been manufactured. Because of this we will focus our attention on the manufacturing process. If this different viewpoint seems to the reader to be a minor point, then it should be pointed out that it changes entirely how an analyst will approach the problem and what techniques he must use to solve it. Instead of a statistical quality control (SQC) problem, we are now faced with a statistical process control (SPC) problem. SQC involves a large amount of inspection. This inspection is usually conducted on final product. However, at this point in the process it is too late to make corrections or adjustments to the process which can correct for the mistakes already made; although, it will certainly be a necessary task if quality is to be improved in future items. SPC methods are based upon the examination of finished *and* semi-finished product at an early stage with some means of rapid and effective feedback. Rapid feedback gives tighter control, saves adding value to defective items, saves time, and reduces the impact of defective material on

scheduling and resulting output. Effective feedback can only be achieved by the use of statistically based process quality control methods [Ref. 6]. SPC methods will allow the quality of the product to be controlled by allowing the process to constantly monitor itself. This means that the operator or technician in charge of the process examines the product as it comes through the process to make sure that it meets the required specifications. He does this by using a sampling procedure which tells him exactly how often to take a sample and how many items to sample. The sampling must of course be conducted randomly.

SPC methods are often very simple when implemented in a manufacturing process. As an example, suppose a quality problem arises within a process. The problem will be noticed reasonably quickly if the process is monitored correctly. After becoming aware of the problem, with minimum delay, the cause is determined and the problem is corrected. The procedure just described is a routine one and occurs frequently in a manufacturing process. The use of SPC methods allows the frequent occurrence of procedures such as the one mentioned above to be dealt with promptly and efficiently.

Problems with quality in a manufacturing process in which SQC procedures are being employed are not always as straight forward. In SQC, a quality problem is sometimes discovered after the product is out of the manufacturer's factory and has been delivered to the customer. If through acceptance sampling procedures, the lot is rejected, then not only has the manufacturer lost money but he has a defective product and it is conceivable that he may not even know why.

The companies that have used SPC procedures over the years have found that quality costs related to SPC are usually known and low. Once SPC procedures are implemented by a company, they tend to remain in use because they are found to be of considerable benefit. Unfortunately, the number of companies that are actually making widespread use of SPC methods in their operation is relatively low when one considers the benefits that SPC methods offer. The major reason found for this low usage is a lack of knowledge, particularly among senior managers. Although these senior managers sometimes recognize quality as being an important part of corporate strategy, they do not appear to know what effective steps to take in order to carry out the strategy. Too often quality is seen as an abstract property and not as a measurable and controllable parameter. [Ref. 6]

B. CONTROL CHARTS AS A MEANS OF PROCESS CONTROL

Assuming we do not want to conduct 100 percent inspection on the process we need to devise a sampling procedure or some other method that will tell us when the quality in our manufacturing process begins to deteriorate. In addition, we would like the method to be more versatile than just being able to tell us when we are producing excessive defective products. We would like the method to be useful in helping us control the process, and a control chart is a widely used statistical method that is well-suited for this task.

The control chart is considered to be a very important tool in statistical process control. Manufacturers readily accept the fact that the measured quality of their product is always subject to a certain amount of random variation which can be attributed to chance. Typically, some inherent random error is present in any process. Variation within the process will, therefore, be inevitable. The control chart is a tool which gives the producer the capability to detect any excessive variation in the process which may be attributed to assignable causes. Such assignable causes may be: differences among machines, materials, or workers, or a number of other factors related to the performance of the process. The reason for the excessive variation is then determined and corrected. Thus, use of the control gives the producer the capability to diagnose and correct many production problems and often brings substantial improvements in product quality. Furthermore, by identifying certain quality variations as chance variations, the control chart tells us when to leave a process alone, thereby preventing unnecessary and frequent adjustments that tend to increase the variability of the process rather than decrease it.

One versatile and widely used control chart is the p-chart. It is the control chart for proportion defective, or fraction defective, which is designated by p. A p-chart works as follows. A sample of a predetermined size is taken at a predetermined interval. The number of items in the sample that are defective, x, is then compared with the sample size, s, to form our ratio $\frac{x}{s}$, or our sample fraction defective which we will refer to as $\bar{p}$. This value is then compared to the control limits on the control chart to get an idea of how the process is performing at the time of the sample being taken. The p-chart can be constructed using data that are already available for other purposes or can readily be made available.

We will use the p-chart in our quality method. Before we can use the p-chart, we must have an estimate of the fraction defective of the process. We will assume that we are able to obtain this estimate without a great deal of difficulty. Now we have nearly all of the information we need to construct the p-chart. Simple statistical calculations

can be used to provide control limits that tell whether assignable causes for variation appear to be present or whether the variations from lot to lot are explainable by means of chance occurrence. We will concern ourselves only with an upper control limit, since a lower control limit on fraction defective is a self-defeating purpose. The upper control limit can be calculated as follows:

$$UCL_{\bar{p}} = \hat{p} + 3\sigma_{\bar{p}} \qquad \text{[Ref. 3].}$$

We should be able to get an estimate for p from past data on the process. We will denote this estimate as $\hat{p}$. We will use the value for $\hat{p}$ as the centerline on the control chart. Also, since, $\bar{p}$ is based on the binomial distribution, we are able to estimate $\sigma_{\bar{p}}$ as follows:

$$\sigma_{\bar{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{s}}$$

As can be seen $\sigma_{\bar{p}}$ is dependent on the sample size, s, which will vary from control chart to control chart. Thus, $\sigma_{\bar{p}}$ will also vary from control chart to control chart.

For illustration purposes, suppose our sampling frequency, m, or the number of items produced between samples being taken, is 1000. Also, assume our sample size, s, is 80 and our estimate $\hat{p}$ of the process fraction defective p is $.015$. We can now calculate $\sigma_{\bar{p}}$ as follows:

$$\sigma_{\bar{p}} = \sqrt{\frac{.015(.985)}{80}}$$
$$= .0136.$$

Now that we have a value for $\sigma_{\bar{p}}$, we can calculate $UCL_{\bar{p}}$, or the upper control limit for the process as follows:

$$UCL_{\bar{p}} = .015 + 3(.0136)$$
$$= .0558$$

We are now able to proceed with the implementation of the control chart for our process. Figure 2 shows a control chart with the parameters defined as follows:

$$\hat{p} = .015,$$
$$\sigma_{\hat{p}} = .0136,$$

and

$$m = 1000.$$

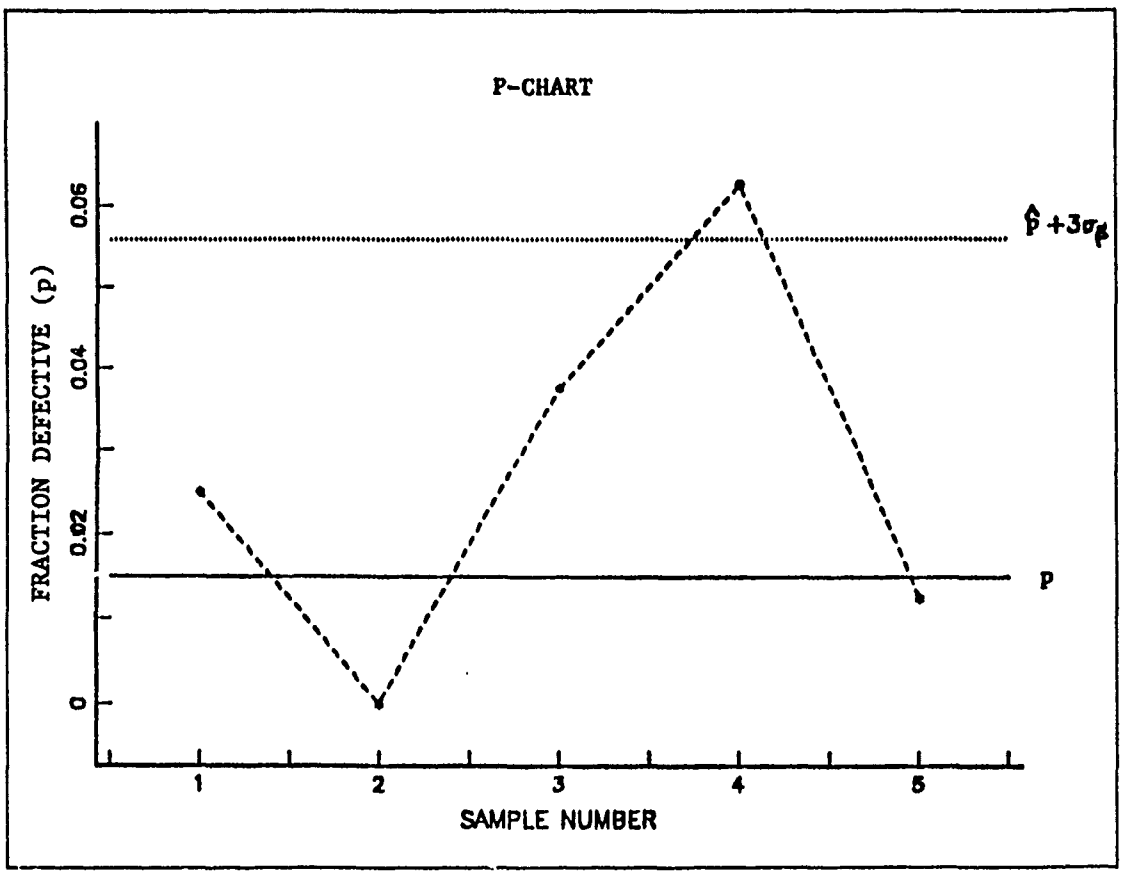

Figure 2. An Example of a p-chart

One should keep in mind that the value for $\sigma_{\hat{p}}$ is heavily dependent upon the sample size, s . What the control chart is telling us in this example is that the process is out of control because it has one point above the upper control limit. As we discussed earlier, what should happen in this instance is that the source of the problem is determined as quickly as possible and then promptly corrected.

C. MEASURING THE EFFECTIVENESS OF CONTROL CHARTS

Taking the concept of control charts one step further, we would like the consumer to be able to determine the effect that using a control chart at the end of a process will have on the quality of the output. The consumer can then judge for himself whether or not acceptance sampling is needed. Once again, the consumer's primary concern is whether or not either procedure will function adequately to furnish product of acceptable quality.

One commonly used measure of control chart performance is the chart's operating characteristic curve or OC curve. A control chart's OC curve gives the probability that (for a given fraction defective) the process is declared under control based on a sample result. Being declared under control is essentially the acceptance of the null hypothesis that the process fraction defective is $\hat{p}$, the value based upon past data of the process. This probability as a function of the current process fraction defective p is denoted $Pa(p)$.

Another well-known measure of control chart performance within quality circles is the average run length curve or ARL curve. The ARL curve gives, for a particular fraction defective, the average number of samples that would have to be taken before the process was determined to be out of control. [Ref. 1] Figure 3 shows an example of a p-chart OC curve and Figure 4 shows an example of an ARL curve.

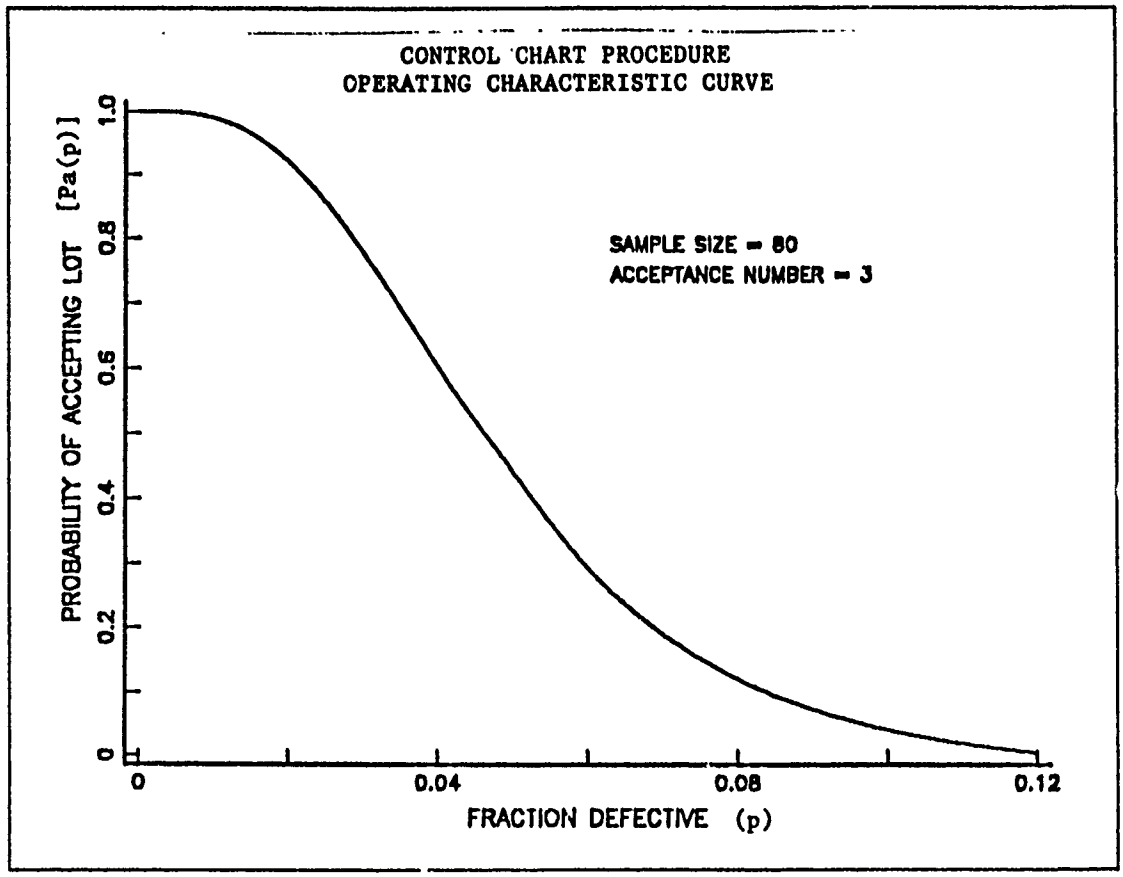

Figure 3. p-chart OC Curve

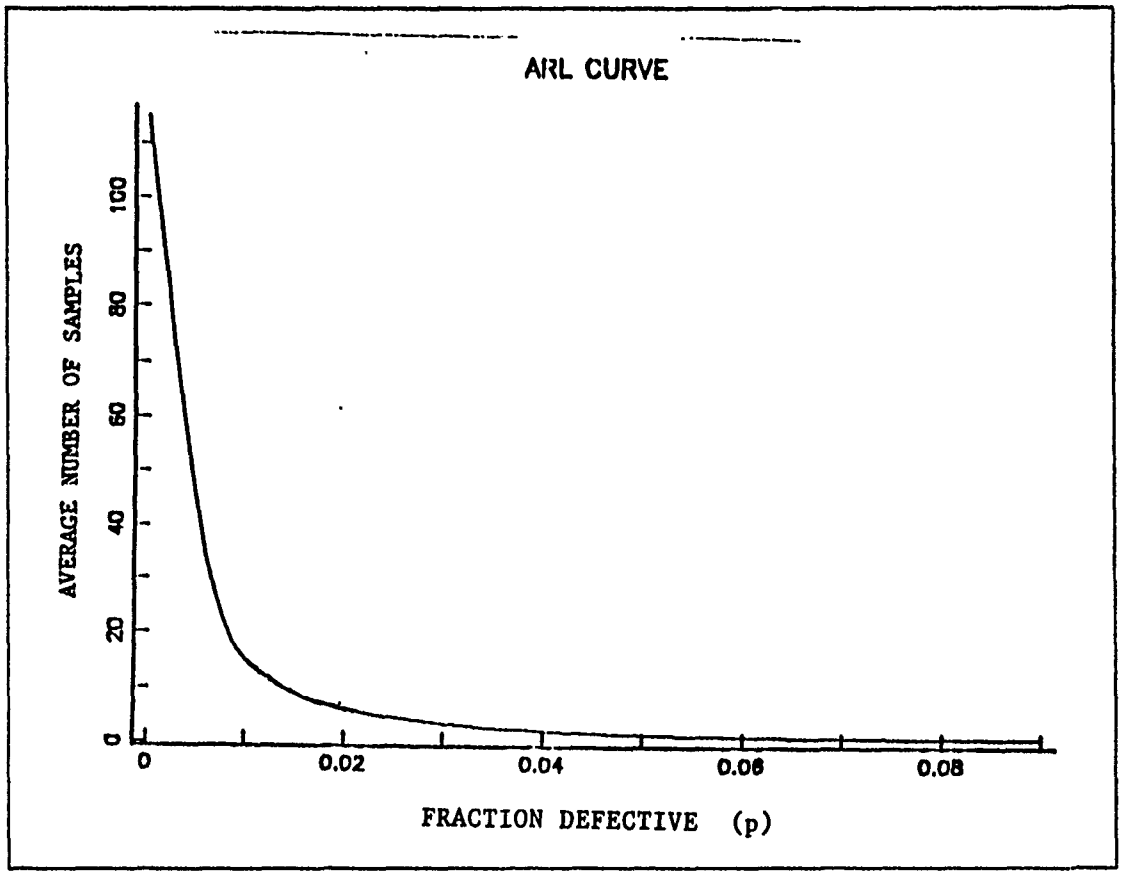

Figure 4. ARL Curve

Both the OC curve and the ARL curve are shown as functions of p , the process fraction defective. Furthermore, there is another less obvious relationship between an ARL curve and an OC curve. It can be shown that the ARL value is the mean of a geometric distribution, and the average run length can be calculated as follows:

$$ARL(p) = \frac{1}{(1 - Pa(p))} . \quad \text{[Ref. 1]}$$

Still, the consumer likely will not be satisfied with either of these methods since neither one will tell him the quality of received goods. The consumer would like to be able to evaluate the producer's method of quality assurance to determine the quality level of the product he is receiving. With respect to the control chart procedure, the

question now becomes: how does the consumer judge a manufacturer's control chart? Upon initial examination of a p-chart one notices the following:

1. The sample size,
2. The sampling interval,
3. The upper control limit, which is a function of the centerline and sample size, and
4. The steps to be taken by the producer in the event the process is determined to be *out of control* , i.e., when the sample proportion is found to be outside the control limits on the control chart.

The customer's concern is that none of above listed factors gives him any direct information about the quality of product that he will receive.

In addition to these factors, it is clear that product quality leaving a final control chart will depend upon how the characteristics which are the primary cause for defects in the process change over time. Clearly, these characteristics will vary from process to process, and in some instances may be unknown or difficult to estimate. In the next section, we will give one simple model of a manufacturing process in which p varies over time. The model will be used to demonstrate the effectiveness of the control chart procedure, and to provide a basis for computing a measure of the quality of goods received by the consumer.

D. A MODEL OF A MANUFACTURING PROCESS

The next step in this approach to our problem is to devise a way of approximating or estimating the performance of the manufacturing process with respect to quality. The simple process we will model is one where a final control chart is employed for overall fraction defective. We shall assume that when the control chart shows that the process is out of control, corrections are made before additional items are produced, and the process fraction defective returns immediately to a lower base value. This base value is the result of inherent, random errors within the process.

When devising this model, one must include the effect that various factors have on the quality of a product. One of these factors is the aging of elements used in the process (machinery, tooling, etc.). Another factor is the random way in which defective products occur and how we record them; e.g., by attributes or by variables. In our model, items are scored by attributes only; that is each item is classified into one of two categories: defective or non-defective.

Another modeling problem that must be addressed is: what happens when the process is determined to be out of control or producing an excessive amount of defective

products. Typically, some form of corrective action is taken to get the process back under control. At times, this corrective action may involve such measures as shutting the process down until the problem can be determined and corrected. Drastic measures such as this are usually not required since the probem can often be corrected without noticeably slowing down production. For modeling purposes, we will assume that when the process is back in control as a result of the necessary corrections being made, the process fraction defective immediately reverts back to a value which is a result of the occurence of inherent, random errors in the process. This seems to be a reasonable assumption when considering the reasons listed above for occurrence of defects in a product. The assumption of the process fraction defective reverting back to its inherent value or state along with the general nature of a manufacturing process allows us to model the process performance as a stochastic process. First, let us suppose we divide up the daily production output quantity of the process into n increments of size m. This can be done on a shift basis instead of a daily basis if the producer so desires. This gives us our sampling frequency. In our example we will arbitrarily choose 6 as the value of n. Thus, we have six production periods per day or per shift and a sample will be taken after every m units of production. Let a be defined as the probability that an item produced by the process when under control is defective due only to the inherent random error within the process. Whenever we have determined the process to be out of control, the model allows the process to become under control again by permitting the probability of producing a defective item to immediately revert back to a. The assumption is made that the problem causing the process to be out of control can be determined and corrected. Also, let us define b as the rate per item produced at which the probability of occurrence of a defective item increases with respect to production. We now have the basis for our process model which is shown in graphical form in Figure 5.

It should be pointed out that in a real world manufacturing process, the parameters a and b will not only be unknown but they will probably be very difficult to estimate. If they were known, then we would have no reason to employ a control chart on the process since we would know when to make the necessary adjustments to the process to keep it producing goods of an acceptable quality. Likewise, we would have no use for a sampling plan if we had available to us known values for a and b. By using our model we hope to have a *rough* means of portraying a change in p over time.

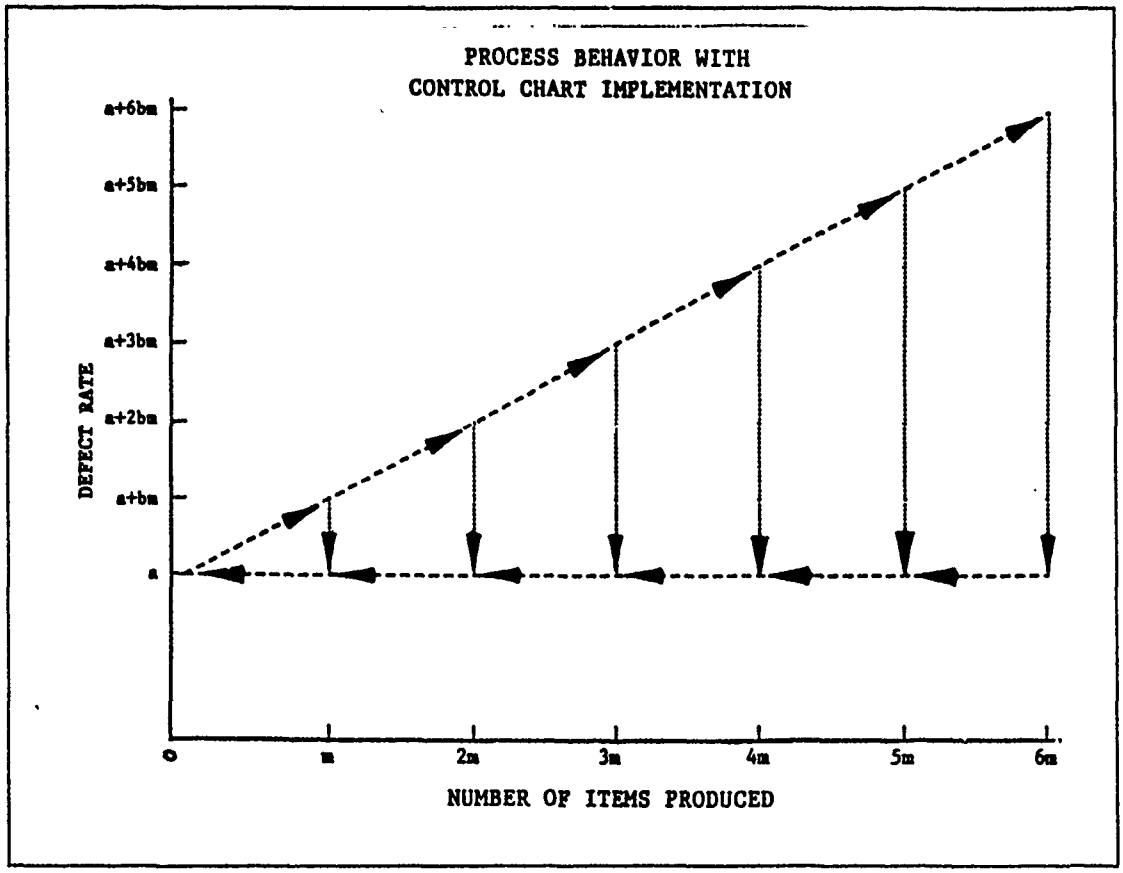

Figure 5. Graphical Representation of Model

In Figure 5 we show the defect rate increasing linearly with the number of items produced. Sampling inspection will take place after every m items are produced. When the process is determined to be out of control via use of the control chart, the defect rate immediately returns to the point designated in the figure by the coordinates $(0, a)$. The process essentially *starts over* at this point. Moving along with the model, let Pa_i equal the probability of concluding that the process is under control at level $a + ibm$ where i is the period number and $a + ibm$ is the process defect rate at the end of period i. The probability Pa_i can be obtained from from the OC curve for the control chart. We can think of im as the number of items produced since the last time the process was determined to be out of control and at which time the probability of producing a defective item was returned to $(0, a)$. If we let P_i equal the fraction

defective for the quantity of goods produced during period i, then we can calculate this value for each period as

$$P_i = a + \frac{2i-1}{2} bm .$$

As we noted earlier, we have *built-in* to our model the control chart decision rules. An important point is that without these decision rules in our model the process fraction defective, p, will increase linearly without bound. This is due to the fact that the control chart is the only measure we are using to decide whether or not the process is under control or not. However, as we indicated earlier the purpose of the model is simply to give us a rough idea of how a manufacturing process performs when a control chart is imposed on it.

E. CALCULATION OF STEADY-STATE PROBABILITIES USING MARKOV CHAIN PROPERTIES

Generally, if and when the manufacturing process reaches a level with defective rate $a + 6bm$ in our example, the probability that we conclude the process is under control will be extremely small. Therefore, we will assume that Pa_6 is equal to zero. This assumption gives us the capability to approximate the behavior of the process with a Markov chain with a finite number of states (six). A Markov chain can describe a process which moves from one state to the next in such a way that the probability of going to one of the next states only depends on the fact that it is presently in a given state and does not depend on any prior states [Ref. 7]. We can summarize what we know thus far in Table 1, which follows.

Table 1. SUMMARY OF MODEL PROBABILITIES

Period or State	Probability of a Defect Occuring at the end of Period i	Probability of Concluding that Process is Under Control at Level $a + ibm$	Fraction Defective in Period i, P_i
1	$a + bm$	Pa_1	$a + \frac{1}{2} bm$
2	$a + 2bm$	Pa_2	$a + \frac{3}{2} bm$
3	$a + 3bm$	Pa_3	$a + \frac{5}{2} bm$
4	$a + 4bm$	Pa_4	$a + \frac{7}{2} bm$
5	$a + 5bm$	Pa_5	$a + \frac{9}{2} bm$
6	$a + 6bm$	0	$a + \frac{11}{2} bm$

The six states in the model correspond to the ends of the six periods listed in Table 1. State 1 is the end of Period 1 where the defect rate is $a + bm$. The Markov chain reaches State 1 as a result of the process being determined to be out of control. When this occurs, corrective action is taken on the process which takes the process defect rate back to a, and then with certainty to $a + bm$ at the end of Period 1.

Referring to Figure 5, one can see the pattern which is a result of the model being a Markov chain. In State 1, the process can only transit to State 2 if it is concluded that the process is under control or to a if it is concluded that the process is out of control. Similarily, in State 2, the process can only transit to State 3 or to a. Let the matrix T, which follows, represent the transition probabilities. This 6 x 6 transition probability matrix shows the probabilities of transitioning to states represented by the columns, given one is in a state represented by one of the six rows. Note that the transition probabilities are either 0, 1.0, or are from the OC curve from the control chart of the process.

$$T = \begin{vmatrix} 1-Pa_1 & Pa_1 & 0 & 0 & 0 & 0 \\ 1-Pa_2 & 0 & Pa_2 & 0 & 0 & 0 \\ 1-Pa_3 & 0 & 0 & Pa_3 & 0 & 0 \\ 1-Pa_4 & 0 & 0 & 0 & Pa_4 & 0 \\ 1-Pa_5 & 0 & 0 & 0 & 0 & Pa_5 \\ 1.0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

For an irreducible ergodic Markov chain, π_j, or the long-run proportion of time that the process is in state j, can be calculated as follows: [Ref. 7]

$$\pi_j = \sum_{i=0}^{\infty} T_{ij}\pi_i$$

and

$$\sum_{j=1}^{\infty} \pi_j = 1 .$$

Thus, for our model, we have

$$\pi_j = \sum_{i=1}^{6} T_{ij}\pi_i , \qquad j = 1,2,...,6 .$$

Using the transition probabilities, we can develop equations that can be used to solve for π_j as follows:

$$\pi_1 = \pi_1(1-Pa_1) + \pi_2(1-Pa_2) + \pi_3(1-Pa_3) + \pi_4(1-Pa_4) + \pi_5(1-Pa_5) + \pi_6. \qquad (3.1)$$

$$\pi_2 = \pi_1 Pa_1 , \qquad (3.2)$$

$$\pi_3 = \pi_2 Pa_2 , \qquad (3.3)$$

$$\pi_4 = \pi_3 Pa_3 , \qquad (3.4)$$

$$\pi_5 = \pi_4 Pa_4 , \tag{3.5}$$

$$\pi_6 = \pi_5 Pa_5 \tag{3.6}$$

and

$$\sum_{j=1}^{6} \pi_j = 1.0. \tag{3.7}$$

We now have seven equations and six unknown variables. (we can take the Pa_i values from the OC curve). We will work with equations (3.2) thru (3.7). If we add these equations together and perform some algebraic manipulation. we end up with the following results:

$$1 - \pi_1 = \pi_1 Pa_1 + \pi_1 Pa_1 Pa_2 + \pi_1 Pa_1 Pa_2 Pa_3 + \pi_1 Pa_1 Pa_2 Pa_3 Pa_4 + \pi_1 Pa_1 Pa_2 Pa_3 Pa_4 Pa_5 ,$$

which simplifies to

$$\pi_1 = \frac{1}{1 + Pa_1 + Pa_1 Pa_2 + Pa_1 Pa_2 Pa_3 + Pa_1 Pa_2 Pa_3 Pa_4 + Pa_1 Pa_2 Pa_3 Pa_4 Pa_5} . \tag{3.8}$$

Equations (3.2) through (3.6) show that each π_i, $(i = 2,...,6)$ is proportional to π_1. We can develop the following equations for π_i :

$$\pi_2 = \frac{Pa_1}{1 + Pa_1 + Pa_1 Pa_2 + Pa_1 Pa_2 Pa_3 + Pa_1 Pa_2 Pa_3 Pa_4 + Pa_1 Pa_2 Pa_3 Pa_4 Pa_5},$$

$$\pi_3 = \frac{Pa_1 Pa_2}{1 + Pa_1 + Pa_1 Pa_2 + Pa_1 Pa_2 Pa_3 + Pa_1 Pa_2 Pa_3 Pa_4 + Pa_1 Pa_2 Pa_3 Pa_4 Pa_5},$$

$$\pi_4 = \frac{Pa_1 Pa_2 Pa_3}{1 + Pa_1 + Pa_1 Pa_2 + Pa_1 Pa_2 Pa_3 + Pa_1 Pa_2 Pa_3 Pa_4 + Pa_1 Pa_2 Pa_3 Pa_4 Pa_5},$$

$$\pi_5 = \frac{Pa_1 Pa_2 Pa_3 Pa_4}{1 + Pa_1 + Pa_1 Pa_2 + Pa_1 Pa_2 Pa_3 + Pa_1 Pa_2 Pa_3 Pa_4 + Pa_1 Pa_2 Pa_3 Pa_4 Pa_5}.$$

and

$$\pi_6 = \frac{Pa_1Pa_2Pa_3Pa_4Pa_5}{1 + Pa_1 + Pa_1Pa_2 + Pa_1Pa_2Pa_3 + Pa_1Pa_2Pa_3Pa_4 + Pa_1Pa_2Pa_3Pa_4Pa_5}.$$

Now that we have determined the long run proportion of time the process spends in each state, and Table 1 shows the fraction defective in each state, we can calculate the fraction defective of the process as determined by the model. We will denote the model's resulting value for the process fraction defective as p'.

We can compute p' as follows:

$$p' = \pi_1(a + \tfrac{1}{2}bm) + \pi_2(a + \tfrac{3}{2}bm) + \pi_3(a + \tfrac{5}{2}bm) + \pi_4(a + \tfrac{7}{2}bm)$$

$$+ \pi_5(a + \tfrac{9}{2}bm) + \pi_6(a + \tfrac{11}{2}bm).$$

If we collect the common terms, we end up with the following equation for fraction defective as computed with the use of our model: [Ref. 7]

$$p' = a + bm \sum_{i=1}^{6} \pi_i \left(\frac{2i - 1}{2} \right). \tag{3.9}$$

The expression for p' in Equation (3.9) provides a way to compute a value for the output fraction defective of the simple process we have modeled. The six π_i values may be obtained from solution of Equation (3.8) for π_i, and successive solution of Equations (3.2) through (3.6) for π_2 through π_6. Solving these equations requires five values: Pa_1, Pa_2, Pa_3, Pa_4, and Pa_5. These values can be obtained from the control chart for the manufacturing process.

An important point to note is that p' may not accurately reflect the level of quality in the product as it ends up in the consumer's warehouse. This is because the model does not consider how the quality level in the product is affected by shipping, onloading,

offloading, etc.. Despite the model's limitations, it gives us a rough estimate of a process' fraction defective when a control chart is employed.

The end result of the model is the value of the manufacturing process' output fraction defective, or more simply put, an estimate of the level of quality in the product produced by the process when there is a control chart at the end. This value p' can be thought of as a means of measuring the effectiveness of the control chart procedure. Furthermore, this value can be compared to the AOQ value of an acceptance sampling procedure to get an idea of which method is more effective. This will be addressed in the next chapter.

F. ANALYZING SOME RESULTS OF AN IMPLEMENTED CONTROL CHART AT THE END OF A PROCESS

It should be clear that the parameters of the model can be changed as necessary in order to estimate the performance of a variety of manufacturing processes. The program listed in Appendix A was written so we would have a means of testing the model with respect to its ability to estimate the output of the process with a control chart. The program allows the user to judge the sensitivity of the model with respect to variation of its parameters a and b, as well as the control chart characteristics $UCL_{\bar{p}}$, sample size s, and sampling interval m. By varying these factors we are able to get a better understanding as to which ones are the most critical in computing the process output average fraction defective, p'.

The program was applied to cases where a varied from 0 to 0.0075, b from 0.000001 to 0.000005, $\hat{p}$ from 0.001 to 0.01, sample size s from 50 to 150, and sampling frequency m from 500 to 2000.

Some results of the computer program (see Appendix B), suggest that, over the range of parameter values used, the estimate of average fraction defective fluctuated the most as the parameters b and m were varied. Hence, b and m or their product appear to have the most influence on the estimate of the output quality of the process. This seems logical when one recalls that m is the sample frequency and b is the rate at which the process performance deteriorates. If a process has a high rate of deterioration, then it is clear that its average fraction defective will be higher as a result even with the use of a control chart. Still, it will be significantly lower when using a control chart than when not using one. Intuitively, this will be true of any manufacturing process in a real world environment and, as explained earlier, it is true by design in our model. This is because our model cannot operate without a control chart. Also, if quality problems

are occurring within a process, the more frequently samples are taken, the greater the frequency of detecting and correcting those problems becomes.

Taking our analysis one step further, we will now focus our attention on a portion of the data. For the cases where the sampling frequency is $m = 1000$ (one third of the data), using the data in Appendix B, we further find that the process fraction defective is most dependent on the process characteristics, a and b, and the sampling frequency m. This was determined by obtaining a linear fit for the data. In particular, the process fraction defective after one period, (a + bm) reflects both the process characteristics and the sampling frequency. Using this as one variable and the control chart rule (UCL) as the other, we applied a least squares fit to our data. The result for the 81 data points where $m = 1000$ was

$$p' = 0.0005 + 1.0094(a + bm) + 0.0733(UCL) .$$

This reflects, for this set of data, the clear influence of process characteristics and sampling frequency. Some values of interest for the $m = 1000$ set of data are shown in Table 2 which follows. Note that this is not a statistical regression analysis, but simply an effort to fit a linear function to a set of data.

Table 2. RESULTS OF LEAST SQUARES FIT TO DATA

	Minimum Value	Maximum Value
$a + bm$	0.001	0.0125
$UCL_{\bar{p}}$	0.0087	0.0522
p'	0.0023	0.019
p from linear fit	0.00212	0.0169
Residual	-0.00416	0.00415

As stated earlier, the purpose of the computer program shown in Appendix A was simply to demonstrate the effectiveness of a final control chart on process output as a function of process parameters and control chart characteristics. It is believed that if

one examines the output, its effectiveness will become apparent. After close examination, one can also get an understanding of the sensitivities of the model to specific parameters. This information can be useful to a quality manager when he is attempting to improve his operation.

Although, the computer program provides a means of evaluating the effectiveness of the control chart procedure, what is really desired is a method of comparing, from the consumer's point of view, reliance on a final control chart to acceptance sampling. This issue is addressed in Chapter IV.

IV. COMPARISON OF THE RELIANCE ON ACCEPTANCE SAMPLING VERSUS A CONTROL CHART PROCEDURE

In this chapter, we will look at various means of comparing acceptance sampling with statistical process control, wherein acceptance sampling would not be employed. First, a qualitative comparison of the corresponding costs of each method will be performed followed by a comparison of the effectiveness of each method.

A. COST COMPARISON

The cost comparison will be approached from the perspective of the base commander. From his viewpoint, any alternative that reduces *his* costs and maintains or increases current levels of effectiveness will attract his interest. Since the proposed alternative may reduce the the number of quality assurance personnel required to be staffed at the base, the base commander will at least reap the amount of savings resulting from this. However, the biggest savings in using a method of quality assurance involving control charts vice acceptance sampling is what results from the elimination of sampling inspection and rectifying inspection.

An outsider may argue that even though in this case the Naval Weapons Support Center is saving money, the overall cost to the Department of Defense is the same. While it is true that there is a shift in cost from the naval base to the defense contractor, there still may be a significant savings involved with the control chart procedure. First of all, the defense contractor should have no need to hire additional personnel when he implements this method. This is because it is necessary and fundamental to the success of the process control method that the technicians and operators on the production line should be the personnel who are carrying out the methods of the control chart procedure. This is where training of personnel is important. It may be necessary to train all of the technicians and machine operators associated with the manufacturing process in the fundamentals of statistical process control and the use of control charts. The major impact of training costs will be felt in the beginning while the changeover to reliance on the control chart procedure is being made. Training will still have to be conducted, though to a lesser degree, on a continual basis. This will be one of the two primary costs of changing the emphasis from an acceptance sampling procedure to a control chart procedure. The other cost is associated with instances when it is necessary to stop the process to determine the reason for the process being out of control. This resulting

downtime in production can be very costly and can quickly cut into the defense contractor's profit margin.

The cost considerations discussed above are difficult to quantify. Why these costs which primarily affect the manufacturer are of concern to the base commander is that he will see them reflected in his contract with the manufacturer. Due to the factors mentioned above, the base commander may see a reduction in his costs when the contract is written if a control chart procedure is used.

B. EFFECTIVENESS COMPARISON

Having concluded that elimination of acceptance sampling may result in a cost savings, we now to focus our attention on the effectiveness of a control chart procedure as opposed to an acceptance sampling procedure. One way of examining the effectiveness of each procedure is to look at the effect of each on the AOQ or average outgoing quality of the process and the quality components.

With the concept of AOQ, we will assume a scheme of 100 percent inspection of rejected lots and that any defective items discovered within the rejected lot will be replaced by good items. Another assumption is that the 100 percent inspection of the rejected lots will ultimately result in 100 percent quality of those lots. Finally, in our analysis, we will assume that the number of defects in our sample is binomially distributed with a mean equal to sp, or the product of the sample size s and lot quality p. We will now show how average outgoing quality is related to p.

Recall that in Chapter II, the AOQ for a process in which a sampling plan and 100 percent rectification are implemented was defined as follows:

$$AOQ = \frac{Pa(p(L-s))}{L} .$$

For illustration purposes, we will use the same MIL-STD 105D sampling plan that we used earlier as an example. Its parameters are as follows:

lot size (L = 1000),
sample size (s = 80),
acceptable quality level (AQL = .015) and
acceptance number (c = 3).

(Note: MIL-STD 105D uses the following letter designations: N for lot size (vice L) and n for sample size (vice s). Different designations are used in this thesis to clarify the difference in the parameters used in the model and the parameters used in MIL-105D.)

Using various values for the quality of incoming product, p, we summarize the resulting AOQ values for the different values of p in Table 3. The values for Pa were taken from the OC curve for this plan shown earlier in Chapter II.

Table 3. AOQ VALUES FOR A MIL-STD 105D SAMPLING PLAN WITH AQL=.015

p	Pa	AOQ
.010	.991	.0091
.015	.966	.0137
.020	.921	.0169
.025	.857	.0197
.030	.779	.0215
.035	.692	.0223
.040	.603	.0222
.045	.515	.0213
.050	.433	.0199
.055	.359	.0182
.060	.294	.0162
.065	.238	.0142
.070	.191	.0123
.075	.151	.0104
.080	.117	.0086

As can be seen the maximum AOQ value is about .0223. This AOQ value is known as the AOQL, or Average Outgoing Quality Limit. The corresponding value of p at which this maximum AOQ occurs is .035. Simply put, the process with use of this par-

ticular sampling plan is ultimately producing its worst level of quality, or AOQL = .0223, when p, the lot quality, is equal to .035. In effect, we can consider this to be the worst case performance of this MIL-STD 105D sampling plan when rectifying inspection is employed.

Under the proposed method of using control charts to gain statistical control of the process, our argument is a very simple one. The process long-run average performance p will simply be equal to the estimate p' of the process average fraction defective obtained from the model. Figure 6 shows the effect of each method on the AOQ of the process.

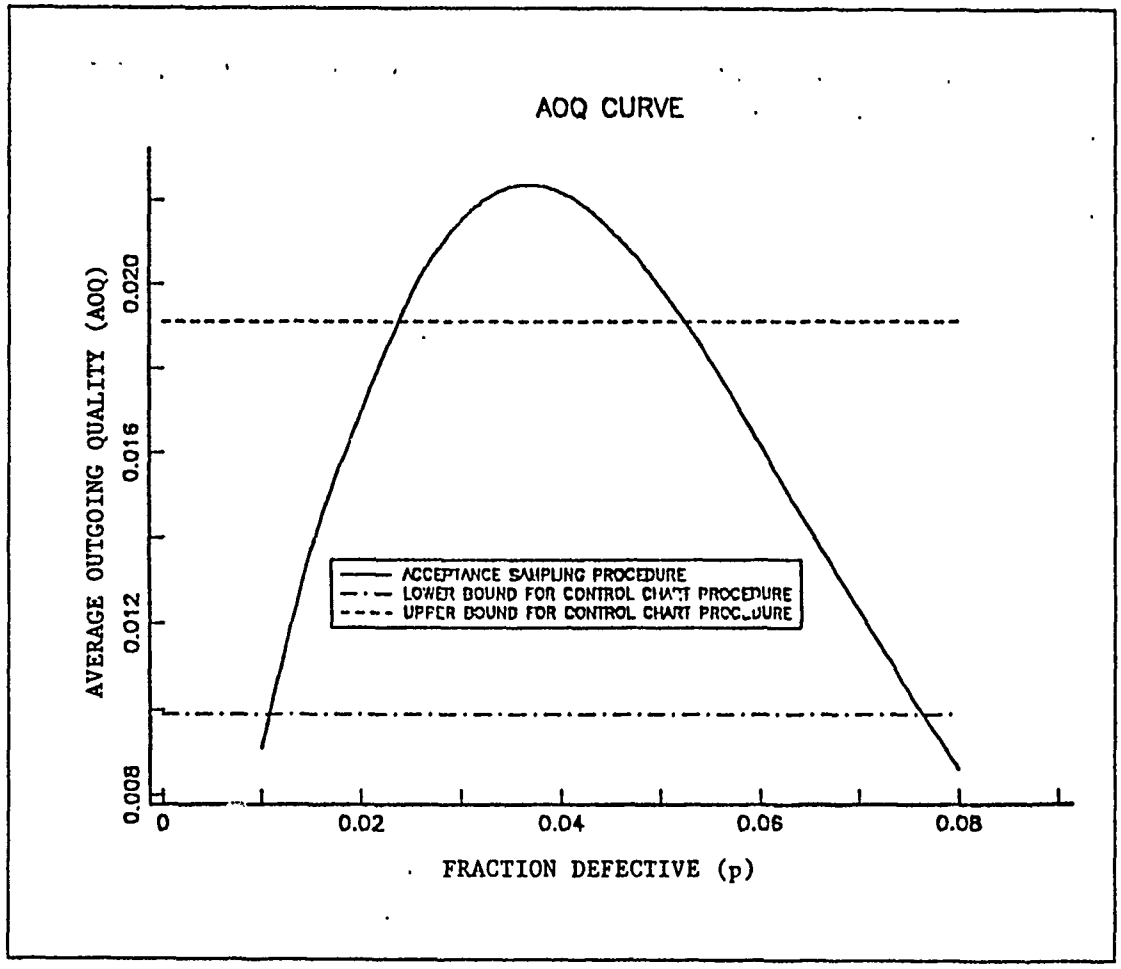

Figure 6. Comparison of the Effect of the Two Methods on AOQ of Process

It can be seen from the figure that at times, MIL-STD 105D performs better and at other times it does worse. In this example the worst case performance is better with

the control chart method. The MIL-STD 105D AOQ curve is for the same sampling plan we have been using as an example throughout this thesis. Once again its parameters are shown below:

$$L = 1000,$$

$$s = 80,$$

and

$$AQL = .015 .$$

The two dashed lines in the figure representing the upper and lower bounds on p' for the proposed method were computed using the following parameters in the model:

$$n = 6,$$

$$m = 1000,$$

$$a = .0075,$$

$b = .000001$ (for the lower bound),

$b = .000005$ (for the upper bound),

$$s = 80,$$

and

$$p = .015 .$$

As we have already established, p' is dependent upon the parameters used in the model. However, in this example the values for the model's parameters were chosen so that the two methods will be operating on processes with roughly the same characteristics. The same value that was used for lot size in the military standard was chosen as

sampling frequency in the model. Also, it was decided it was best to use the same sample size in each method. The inherent probability that the process will produce a defective item a was arbitrarily chosen to be half of the AQL value. It is believed that for most manufacturing processes this is an overstatement of the tendency for the process to produce a defective item. The rate at which this probability of producing a defective item increases with time b was chosen to take on two values for illustrations purposes due to its uncertainty. The lower bound value represents what is believed to be a reasonable deterioration rate for the process; whereas, the upper bound represents an approximation of a worst case scenario. It is difficult to compare the two methods on a equal basis; however, it is believed that the estimates made here are conservative and provide for a rough, but reasonable comparison of the two methods.

Hence, we conclude that by using this means of comparison, it is reasonably evident that implementation of the proposed method of reliance on a control chart procedure should be given consideration because it will likely result in a cost savings. Also, our analysis has shown that it some cases it can be at least as effective as the current method of acceptance sampling which is being used.

In the following chapter, we will summarize our study and make further conclusions with regard to what we believe the effect of a control chart procedure similar to the one presented here will have on a manufacturing process if implemented.

V. SUMMARY AND SUGGESTIONS FOR FURTHER STUDY

The purpose of this thesis was to conduct research into the area of quality assurance and to determine whether or not there might exist a feasible alternative to ensuring quality in military systems as compared to the methods being used today. Specifically, the Commanding Officer of the Naval Weapons Support Center in Crane, Indiana, would like to know if it would be feasible for him to rely only on the manufacturer's operation and quality assurance methods to ensure an acceptable level of quality in the product he is receiving from the manufacturer, and how he could evaluate these methods. Since there seems to be a slow but sure shift in the manufacturing industry towards statistical process control, it was decided that this thesis would concentrate on an alternative of this nature.

Acceptance sampling conducted by the consumer requires that the military agency staff a group of personnel knowledgeable in quality assurance methods. The method presented in this thesis, consistent with the fundamental concept of statistical process control, shifts the emphasis of product quality back to the manufacturer where ideally problems in quality are detected early and corrected on the spot. In order for the customer to shift from the reliance on acceptance sampling to reliance on control charts at the manufacturing operation, is necessary to show that upon consideration of the producer's quality program, the product he will receive without acceptance sampling is satisfactory. To do this, an effective measure is needed to compare the alternatives of acceptance sampling and control charts.

As a way of measuring the effectiveness of the control chart on the output of a manufacturing operation, a model of a manufacturing process was developed. The primary assumption made in constructing the model was that a linear deterioration rate over time existed with respect to quality. Another assumption made was that when the process is determined to be out of control, the problem can be determined and corrected. Upon correcting the problem, the value for the rate at which defects occur returns immediately to a value that is due only to the random error present within the process. The model requires inputs for sample frequency; sample size; the probability of a defect occurring due to inherent or random error present within the process; the rate at which the probability of the occurance of a defect increases with time; and the centerline value for the control chart to be used. The output p' of the model is an estimate of the

process average output fraction defective when the control chart is in use. The paramters in the model that were determined to be the most influential in this estimate were the sample frequency and the rate at which the probability of an occurence of a defective product increased with respect to time.

Although, the main purpose of the model is to show one way to evaluate the effectiveness of a control chart, a quality manager may be able to use it as a decision aid if control charts are being employed in his operation and if he believes his manufacturing process possesses the necessary characteristics (i.e., linear deterioration rate, etc.). The manager will have to be able to estimate within reasonable accuracy the parameters in the model as they apply to his process in order for the model to provide him with useful information. The quality manager will be able to derive his estimates from existing data concerning past performance of the process. Even if useful data does exist on the performance of the process, the parameters in question will probably be difficult to estimate due to the uncertainty of their behavior, as will probably be borne out by the data. Clearly, if the manager were to have at his disposal estimates of these parameters which involved little or no uncertainty then he would have no use for a control chart to begin with. Nonetheless, if the quality manager is able to obtain estimates of a and b then he may be able to use the model as a decision aid to assist him in improving the efficiency of his process.

In Chapter IV, the argument was made that even if reliance on a control chart without acceptance sampling resulted in approximately the same level of quality in the product, it would result in a cost savings because it would reduce the need for staffing of quality assurance personnel by the customer. We also argued that the proposed procedure would reduce costs because a manufacturer is much better equipped to perform a quality assurance operation than the consumer. For statistical process control to be effective, the quality assurance methods *must* be performed by the manufacturer.

There is much room for improvement in manufacturing methodologies and quality assurance techniques. It is recommended that this research be taken further by approaching it slightly differently. An alternative approach to the problem is to model the manufacturing process using two rates of average fraction defective and the time between the process changing between the two would be exponential. There are many manufacturing systems that would lend themselves to this type of model.

It is sincerely hoped that the methods and procedures presented in this thesis will be useful in helping the Naval Weapons Support Center and other Department of Defense agencies in dealing with quality assurance issues.

APPENDIX A.

Program to Generate OC Curve Probabilities

```
      INTEGER M(1:10),MM,N,S(1:10),SS,AA,BB,C,ICOUNT,PP,NLINE
      REAL PA(1:10),PI(1:10),FRADEF(1:10),SUM,A(1:10),B(1:10)
      REAL PTOTAL,CL(1:10),UCL(1:10,1:10),DENOM(1:10),DSUM
      REAL PPRIME(1:10,1:10,1:10,1:10,1:10)
*
*  * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *  *
*  ////////////////////////////////////////////////////////////////////////////  *
*  //                                                                        //  *
*  //    The purpose of this program is to demonstrate the                   //  *
*  //    effectiveness of a control chart procedure when it is               //  *
*  //    imposed upon a manufacturing process.  This is done by              //  *
*  //    evaluating a model of the process.  Given the necessary             //  *
*  //    inputs, the OC curve probabilities are calculated.                  //  *
*  //    Since, the process is modeled via the use of a markov               //  *
*  //    chain, the steady-state probabilities are then                      //  *
*  //    calculated.  These calculations allow for the                       //  *
*  //    computation of an estimate of a fraction defective                  //  *
*  //    for the process in question.                                        //  *
*  //                                                                        //  *
*  //                                                                        //  *
*  ////////////////////////////////////////////////////////////////////////////  *
*  * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *  *
*
C
C
C   Prompt the user for data entry.
C
      PRINT *, 'To check the sensitivity of the model to each parameter'
      PRINT *, 'enter the number of different values you would like to'
      PRINT *, 'enter for each parameter.'
      PRINT *
      READ *, NUM
      PRINT *
      PRINT *, 'Enter the value for the number of periods,  n.'
      PRINT *, 'This value will not vary.  To change it you must'
      PRINT *, 'run the program again.'
      PRINT *
      READ *, N
      PRINT *
      DO 5 I = 1,NUM
         PRINT *
         PRINT *, 'Enter the value for the sampling frequency,  m.'
         PRINT *
         READ *, M(I)
         PRINT *
         PRINT *, 'Enter the value for the sample size,  s.'
         PRINT *
```

```
      READ *, S(I)
      PRINT *
      PRINT *, 'Enter the value for  a,  the probability the process'
      PRINT *, 'will produce a defective item when it is performi g'
      PRINT *, 'at peak efficiency.'
      PRINT *
      READ *, A(I)
      PRINT *
      PRINT *, 'Enter the value for  B  which is the rate at which'
      PRINT *, 'the probabilty of producing a defective item'
      PRINT *, 'increases with respect to time.'
      PRINT *
      READ *, B(I)
      PRINT *
      PRINT *, 'Enter the value for the centerline on the control'
      PRINT *, 'chart.'
      PRINT *
      READ *, CL(I)
      PRINT *
    5 CONTINUE
      ICOUNT = 0
      NLINE = 0
      DO 60 MM = 1,NUM
        DO 55 SS = 1,NUM
          DO 50 AA = 1,NUM
            DO 45 BB = 1,NUM
              DO 40 PP = 1,NUM
C
C
C   Calculate the upper control limit for the control chart
C
              UCL(PP,SS) = CL(PP) + 3 * SQRT(CL(PP) * (1 - CL(PP))
     +                   / S(SS))
              C = S(SS) * UCL(PP,SS)
C
C
C   Use the subroutine POISS to calculate the OC curve probabilities.
C
              DO 10 I = 1,N
                CALL POISS(A,B,I,M,S,PA,AA,BB,MM,SS,C)
   10         CONTINUE
C
C
C   Calculate the steady-state probabilities.
C
              PTOTAL = 0
              DSUM = 1.0
              DO 20 J = N,2,-1
                 DENOM(J) = 1.0
                 DO 15 I = J-1,1,-1
                    DENOM(J) = DENOM(J) * PA(I)
   15            CONTINUE
                 DSUM = DSUM + DENOM(J)
   20         CONTINUE
              PI(1)  =  1 / DSUM
              PTOTAL = PTOTAL + PI(1)
```

```
                DO 25 I = 2,N
                   PI(I) = PI(I-1) * PA(I-1)
                   PTOTAL = PTOTAL + PI(I)
   25           CONTINUE
C
C
C   Scale the probabilities to ensure that they sum to one.
C
                DO 30 I = 1,N
                   PI(I) = PI(I) / PTOTAL
   30           CONTINUE
C
C
C   Calculate the fraction defective for each period.
C
                SUM = 0
                DO 35 I = 1,N
                   FRADEF(I) = PI(I) * ((2*I - 1) / 2)
                   SUM = SUM + FRADEF(I)
   35           CONTINUE
C
C
C   Calculate the average fraction defective for the process.
C
                PPRIME(PP,BB,AA,SS,MM) = A(AA) + (B(BB) * M(MM) * SUM)
   40         CONTINUE
   45       CONTINUE
   50     CONTINUE
   55   CONTINUE
   60 CONTINUE
C
C
C  Print out a definition list of the parameters.
C
      WRITE (4,65)
   65 FORMAT ('1',/,15X,'Definition of parameters',
     +/,1X,'m =   sampling frequency',/,1X,'s =   sample size',/,1X,
     +'a =   inherent probability of the process producing a defective',
     +' item',/,1X,'b =   rate at which the probabilitiy of producing',
     +' a defective',/,9X,'item increases over time',/,1X,'CL  =',
     +' desired centerline value on control chart',/,1X,'UCL = ',
     +'computed upper control limit for control chart',/,1X,'p   = ',
     +'resulting estimate of the process average fraction defective'//)
C
C
C  Print a summary of parameter values for which the program was run.
C
      WRITE (4,70) NUM,N
   70 FORMAT (10X,'The number of different values entered for each',/,
     +10X,'parameter (excluding  n) was: ',I1,/,10X,'The value entered',
     +' for  n  was: ',I2)
      DO 80 I = 1,NUM
        WRITE (4,75) M(I),S(I),A(I),B(I),CL(I)
   75   FORMAT (2X,'The following parameter values were entered:',/,
     +          5X,'M = ',I4,4X,'S = ',I3,4X,'A = ',F5.4,4X,'B = ',
     +          F7.6,4X,'CL = ',F5.4)
```

```
   80 CONTINUE
C
C
C  Printout column headings for table.
C
      WRITE (4,85)
   85 FORMAT (2X,'Case',5X,'m',7X,'s',6X,'a',9X,'b',9X,
     +  'CL',7X,'UCL',7X,'p'/,2X,'----',2X,'----',5X,
     +  '---',3X,'-----',3X,'---------',4X,'----',3X,'-------',7X,
     +  '-------'/)
C
C
C   Print out all calculated values.
C
      DO 120 MM = 1,NUM
        DO 115 SS = 1,NUM
          DO 110 AA = 1,NUM
            DO 105 BB = 1,NUM
              DO 100 PP = 1,NUM
                ICOUNT = ICOUNT + 1
                NLINE = NLINE + 1
                IF (NLINE .LT. 40) GOTO 90
                WRITE (4,65)
                WRITE (4,85)
                NLINE = 0
   90           WRITE (4,95) ICOUNT,M(MM),S(SS),A(AA),B(BB),CL(PP),
     +                 UCL(PP,SS),PPRIME(PP,BB,AA,SS,MM)
   95           FORMAT (2X,I4,2X,I5,4X,I4,3X,F5.4,3X,
     +                 F9.8,3X,F5.4,3X,F5.4,3X,F7.6)
  100         CONTINUE
  105       CONTINUE
  110     CONTINUE
  115   CONTINUE
  120 CONTINUE
      STOP
      END
      SUBROUTINE POISS(A,B,I,M,S,PA,AA,BB,MM,SS,C)
      INTEGER K,M(1:10),S(1:10),MM,SS,AA,BB,C
      REAL*4 ERROR,TOTAL,EPSLON,MU(1:10),PA(1:10),A(1:10),B(1:10)
C
*  ------------------------------------------------------------  *
*  ============================================================  *
*  ////////////////////////////////////////////////////////////  *
*  //                                                        //  *
*  //   This subprogram will compute poisson probabilities   //  *
*  //   by making use of the following equation:             //  *
*  //                                                        //  *
*  //             P(X = K) = ((EXP(-MU)) * (MU**K))/K ! ,    //  *
*  //                                                        //  *
*  //   Where MU = a parameter based on the sample size and  //  *
*  //              process quality.                          //  *
*  //                                                        //  *
*  ////////////////////////////////////////////////////////////  *
*  ============================================================  *
*
C
```

```
      MU(I) = S(SS) * (A(AA) + (I-1) * B(BB) * M(MM))
C
C
C  Initialize variables
C
      K = 0
C
C  When K = 0, to avoid division by 0, the 'prob' for K = 0 is simply
C  EXP(-MU)
C
      PA(I) = EXP(-MU(I))
      TOTAL = 0.
C
C
C  Begin summing the probabilities to find the cumulative probability.
C
      TOTAL = TOTAL + PA(I)
 5    IF ((K .GE. C ) .OR. (TOTAL .GT. .99999)) GOTO 10
      K = K + 1
C
C
C  Calculate the probability of exactly  K  occurrences.
C
      PA(I)= (PA(I) * MU(I)) / K
C
C
C  Continue summing the probabilities to find the cumulative probability
C
      TOTAL = TOTAL + PA(I)
      GOTO 5
C
C
C  Set Pa(i) equal to the cumulative probability and return to main
C  program
C
10    PA(I) = TOTAL
      END
```

APPENDIX B.

Results of Computer Program

DEFINITION OF PARAMETERS

m = SAMPLING FREQUENCY
s = SAMPLE SIZE
a = INHERENT PROBABILITY OF THE PROCESS PRODUCING A DEFECTIVE ITEM
b = RATE AT WHICH THE PROBABILITY OF PRODUCING A DEFECTIVE ITEM INCREASES OVER TIME
c1 = DESIRED CENTERLINE VALUE ON CONTROL CHART (ROUGH ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE)
UCL = COMPUTED UPPER CONTROL LIMIT FOR CONTROL CHART
p = RESULTING ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE

THE NUMBER OF DIFFERENT VALUES ENTERED FOR EACH PARAMETER (EXCLUDING n) WAS: 3
THE VALUE ENTERED FOR n WAS: 6

THE FOLLOWING PARAMETER VALUES WERE ENTERED:
m = 500 s = 50 a = .0000 b = .000001 c1 = .0010

THE FOLLOWING PARAMETER VALUES WERE ENTERED:
m = 1000 s = 100 a = .0050 b = .000002 c1 = .0025

THE FOLLOWING PARAMETER VALUES WERE ENTERED:
m = 2000 s = 150 a = .0075 b = .000005 c1 = .0100

Case	m	s	a	b	c1	UCL	p
1	500	50	.0000	.000001	.0010	.0144	.0012
2	500	50	.0000	.000001	.0025	.0237	.0012
3	500	50	.0000	.000001	.0100	.0522	.0012
4	500	50	.0000	.000002	.0010	.0144	.0022
5	500	50	.0000	.000002	.0025	.0237	.0025
6	500	50	.0000	.000002	.0100	.0522	.0025
7	500	50	.0000	.000005	.0010	.0144	.0047
8	500	50	.0000	.000005	.0025	.0237	.0060
9	500	50	.0000	.000005	.0100	.0522	.0062
10	500	50	.0050	.000001	.0010	.0144	.0058
11	500	50	.0050	.000001	.0025	.0237	.0062
12	500	50	.0050	.000001	.0100	.0522	.0062

13	500	50	.0050	.000002	.0010	.0144	.0066
14	500	50	.0050	.000002	.0025	.0237	.0074
15	500	50	.0050	.000002	.0100	.0522	.0075
16	500	50	.0050	.000005	.0010	.0144	.0084
17	500	50	.0050	.000005	.0025	.0237	.0106
18	500	50	.0050	.000005	.0100	.0522	.0111
19	500	50	.0075	.000001	.0010	.0144	.0082
20	500	50	.0075	.000001	.0025	.0237	.0086
21	500	50	.0075	.000001	.0100	.0522	.0087
22	500	50	.0075	.000002	.0010	.0144	.0088
23	500	50	.0075	.000002	.0025	.0237	.0098
24	500	50	.0075	.000002	.0100	.0522	.0100
25	500	50	.0075	.000005	.0010	.0144	.0104
26	500	50	.0075	.000005	.0025	.0237	.0128
27	500	50	.0075	.000005	.0100	.0522	.0135
28	500	100	.0000	.000001	.0010	.0105	.0012
29	500	100	.0000	.000001	.0025	.0175	.0012
30	500	100	.0000	.000001	.0100	.0398	.0012
31	500	100	.0000	.000002	.0010	.0105	.0024
32	500	100	.0000	.000002	.0025	.0175	.0024
33	500	100	.0000	.000002	.0100	.0398	.0025
34	500	100	.0000	.000005	.0010	.0105	.0055
35	500	100	.0000	.000005	.0025	.0175	.0055
36	500	100	.0000	.000005	.0100	.0398	.0062
37	500	100	.0050	.000001	.0010	.0105	.0061
38	500	100	.0050	.000001	.0025	.0175	.0061

DEFINITION OF PARAMETERS

m = SAMPLING FREQUENCY
s = SAMPLE SIZE
a = INHERENT PROBABILITY OF THE PROCESS PRODUCING A DEFECTIVE ITEM
b = RATE AT WHICH THE PROBABILITY OF PRODUCING A DEFECTIVE ITEM INCREASES OVER TIME
cl = DESIRED CENTERLINE VALUE ON CONTROL CHART (ROUGH ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE)
UCL = COMPUTED UPPER CONTROL LIMIT FOR CONTROL CHART
p = RESULTING ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE

Case	m	s	a	b	cl	UCL	p
39	500	100	.0050	.000001	.0100	.0398	.0062
40	500	100	.0050	.000002	.0010	.0105	.0070
41	500	100	.0050	.000002	.0025	.0175	.0070
42	500	100	.0050	.000002	.0100	.0398	.0075
43	500	100	.0050	.000005	.0010	.0105	.0093
44	500	100	.0050	.000005	.0025	.0175	.0093
45	500	100	.0050	.000005	.0100	.0398	.0111
46	500	100	.0075	.000001	.0010	.0105	.0084
47	500	100	.0075	.000001	.0025	.0175	.0084
48	500	100	.0075	.000001	.0100	.0398	.0087
49	500	100	.0075	.000002	.0010	.0105	.0092
50	500	100	.0075	.000002	.0025	.0175	.0092
51	500	100	.0075	.000002	.0100	.0398	.0100
52	500	100	.0075	.000005	.0010	.0105	.0112
53	500	100	.0075	.000005	.0025	.0175	.0112
54	500	100	.0075	.000005	.0100	.0398	.0134
55	500	150	.0000	.000001	.0010	.0087	.0012
56	500	150	.0000	.000001	.0025	.0147	.0012
57	500	150	.0000	.000001	.0100	.0344	.0012
58	500	150	.0000	.000002	.0010	.0087	.0024
59	500	150	.0000	.000002	.0025	.0147	.0025
60	500	150	.0000	.000002	.0100	.0344	.0025
61	500	150	.0000	.000005	.0010	.0087	.0049
62	500	150	.0000	.000005	.0025	.0147	.0058
63	500	150	.0000	.000005	.0100	.0344	.0062
64	500	150	.0050	.000001	.0010	.0087	.0059
65	500	150	.0050	.000001	.0025	.0147	.0062
66	500	150	.0050	.000001	.0100	.0344	.0062
67	500	150	.0050	.000002	.0010	.0087	.0066
68	500	150	.0050	.000002	.0025	.0147	.0072
69	500	150	.0050	.000002	.0100	.0344	.0075
70	500	150	.0050	.000005	.0010	.0087	.0082
71	500	150	.0050	.000005	.0025	.0147	.0098
72	500	150	.0050	.000005	.0100	.0344	.0112
73	500	150	.0075	.000001	.0010	.0087	.0082
74	500	150	.0075	.000001	.0025	.0147	.0085
75	500	150	.0075	.000001	.0100	.0344	.0087
76	500	150	.0075	.000002	.0010	.0087	.0087
77	500	150	.0075	.000002	.0025	.0147	.0095

DEFINITION OF PARAMETERS

n = SAMPLING FREQUENCY
s = SAMPLE SIZE
a = INHERENT PROBABILITY OF THE PROCESS PRODUCING A DEFECTIVE ITEM
b = RATE AT WHICH THE PROBABILITY OF PRODUCING A DEFECTIVE ITEM INCREASES OVER TIME
cl = DESIRED CENTERLINE VALUE ON CONTROL CHART (ROUGH ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE)
UCL = COMPUTED UPPER CONTROL LIMIT FOR CONTROL CHART
p = RESULTING ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE

Case	m	s	a	b	cl	UCL	p
78	500	150	.0075	.000002	.0100	.0344	.0100
79	500	150	.0075	.000005	.0010	.0087	.0100
80	500	150	.0075	.000005	.0025	.0147	.0116
81	500	150	.0075	.000005	.0100	.0344	.0136
82	1000	50	.0000	.000001	.0010	.0144	.0022
83	1000	50	.0000	.000001	.0025	.0237	.0025
84	1000	50	.0000	.000001	.0100	.0522	.0025
85	1000	50	.0000	.000002	.0010	.0144	.0040
86	1000	50	.0000	.000002	.0025	.0237	.0049
87	1000	50	.0000	.000002	.0100	.0522	.0050
88	1000	50	.0000	.000005	.0010	.0144	.0074
89	1000	50	.0000	.000005	.0025	.0237	.0109
90	1000	50	.0000	.000005	.0100	.0522	.0121
91	1000	50	.0050	.000001	.0010	.0144	.0066
92	1000	50	.0050	.000001	.0025	.0237	.0074
93	1000	50	.0050	.000001	.0100	.0522	.0075
94	1000	50	.0050	.000002	.0010	.0144	.0079
95	1000	50	.0050	.000002	.0025	.0237	.0095
96	1000	50	.0050	.000002	.0100	.0522	.0099
97	1000	50	.0050	.000005	.0010	.0144	.0105
98	1000	50	.0050	.000005	.0025	.0237	.0149
99	1000	50	.0050	.000005	.0100	.0522	.0168
100	1000	50	.0075	.000001	.0010	.0144	.0088
101	1000	50	.0075	.000001	.0025	.0237	.0098
102	1000	50	.0075	.000001	.0100	.0522	.0100
103	1000	50	.0075	.000002	.0010	.0144	.0099
104	1000	50	.0075	.000002	.0025	.0237	.0118
105	1000	50	.0075	.000002	.0100	.0522	.0124
106	1000	50	.0075	.000005	.0010	.0144	.0122
107	1000	50	.0075	.000005	.0025	.0237	.0168
108	1000	50	.0075	.000005	.0100	.0522	.0190
109	1000	100	.0000	.000001	.0010	.0105	.0024
110	1000	100	.0000	.000001	.0025	.0175	.0024
111	1000	100	.0000	.000001	.0100	.0398	.0025
112	1000	100	.0000	.000002	.0010	.0105	.0046
113	1000	100	.0000	.000002	.0025	.0175	.0046
114	1000	100	.0000	.000002	.0100	.0398	.0050
115	1000	100	.0000	.000005	.0010	.0105	.0086
116	1000	100	.0000	.000005	.0025	.0175	.0086

DEFINITION OF PARAMETERS

m = SAMPLING FREQUENCY
s = SAMPLE SIZE
a = INHERENT PROBABILITY OF THE PROCESS PRODUCING A DEFECTIVE ITEM
b = RATE AT WHICH THE PROBABILITY OF PRODUCING A DEFECTIVE ITEM INCREASES OVER TIME
cl = DESIRED CENTERLINE VALUE ON CONTROL CHART (ROUGH ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE)
UCL = COMPUTED UPPER CONTROL LIMIT FOR CONTROL CHART
p = RESULTING ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE

Case	m	s	a	b	cl	UCL	p
117	1000	100	.0000	.000005	.0100	.0398	.0119
118	1000	100	.0050	.000001	.0010	.0105	.0070
119	1000	100	.0050	.000001	.002	.0175	.0070
120	1000	100	.0050	.[illegible]0001	.0100	.0398	.0075
121	1000	100	.0050	.0[illegible]0002	.0010	.0105	.0086
122	1000	100	.0050	.000002	.0025	.0175	.0086
123	1000	100	.0050	.000002	.0100	.0398	.0099
124	1000	100	.0050	.000005	.0010	.0105	.0117
125	1000	100	.0050	.000005	.0025	.0175	.0117
126	1000	100	.0050	.000005	.0100	.0398	.0162
127	1000	100	.0075	.000001	.0010	.0105	.0092
128	1000	100	.0075	.000001	.0025	.0175	.0092
129	1000	100	.0075	.000001	.0100	.0398	.0100
130	1000	100	.0075	.000002	.0010	.0105	.0106
131	1000	100	.0075	.000002	.0025	.0175	.0106
132	1000	100	.0075	.000002	.0100	.0398	.0123
133	1000	100	.0075	.000005	.0010	.0105	.0132
134	1000	100	.0075	.000005	.0025	.0175	.0132
135	1000	100	.0075	.000005	.0100	.0398	.0182
136	1000	150	.0000	.000001	.0010	.0087	.0024
137	1000	150	.0000	.000001	.0025	.0147	.0025
138	1000	150	.0000	.000001	.0100	.0344	.0025
139	1000	150	.0000	.000002	.0010	.0087	.0042
140	1000	150	.0000	.000002	.0025	.0147	.0048
141	1000	150	.0000	.000002	.0100	.0344	.0050
142	1000	150	.0000	.000005	.0010	.0087	.0069
143	1000	150	.0000	.000005	.0025	.0147	.0092
144	1000	150	.0000	.000005	.0100	.0344	.0122
145	1000	150	.0050	.000001	.0010	.0087	.0066
146	1000	150	.0050	.000001	.0025	.0147	.0072
147	1000	150	.0050	.000001	.0100	.0344	.0075
148	1000	150	.0050	.000002	.0010	.0087	.0078
149	1000	150	.0050	.000002	.0025	.0147	.0090
150	1000	150	.0050	.000002	.0100	.0344	.0100
151	1000	150	.0050	.000005	.0010	.0087	.0098
152	1000	150	.0050	.000005	.0025	.0147	.0122
153	1000	150	.0050	.000005	.0100	.0344	.0167
154	1000	150	.0075	.000001	.0010	.0087	.0087
155	1000	150	.0075	.000001	.0025	.0147	.0095

DEFINITION OF PARAMETERS

m = SAMPLING FREQUENCY
s = SAMPLE SIZE
a = INHERENT PROBABILITY OF THE PROCESS PRODUCING A DEFECTIVE ITEM
b = RATE AT WHICH THE PROBABILITY OF PRODUCING A DEFECTIVE ITEM INCREASES OVER TIME
cl = DESIRED CENTERLINE VALUE ON CONTROL CHART (ROUGH ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE)
UCL = COMPUTED UPPER CONTROL LIMIT FOR CONTROL CHART
p = RESULTING ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE

Case	m	s	a	b	cl	UCL	p
156	1000	150	.0075	.000001	.0100	.0344	.0100
157	1000	150	.0075	.000002	.0010	.0087	.0096
158	1000	150	.0075	.000002	.0025	.0147	.0110
159	1000	150	.0075	.000002	.0100	.0344	.0124
160	1000	150	.0075	.000005	.0010	.0087	.0113
161	1000	150	.0075	.000005	.0025	.0147	.0137
162	1000	150	.0075	.000005	.0100	.0344	.0189
163	2000	50	.0000	.000001	.0010	.0144	.0040
164	2000	50	.0000	.000001	.0025	.0237	.0049
165	2000	50	.0000	.000001	.0100	.0522	.0050
166	2000	50	.0000	.000002	.0010	.0144	.0065
167	2000	50	.0000	.000002	.0025	.0237	.0091
168	2000	50	.0000	.000002	.0100	.0522	.0098
169	2000	50	.0000	.000005	.0010	.0144	.0108
170	2000	50	.0000	.000005	.0025	.0237	.0172
171	2000	50	.0000	.000005	.0100	.0522	.0217
172	2000	50	.0050	.000001	.0010	.0144	.0079
173	2000	50	.0050	.000001	.0025	.0237	.0095
174	2000	50	.0050	.000001	.0100	.0522	.0099
175	2000	50	.0050	.000002	.0010	.0144	.0097
176	2000	50	.0050	.000002	.0025	.0237	.0133
177	2000	50	.0050	.000002	.0100	.0522	.0146
178	2000	50	.0050	.000005	.0010	.0144	.0134
179	2000	50	.0050	.000005	.0025	.0237	.0203
180	2000	50	.0050	.000005	.0100	.0522	.0254
181	2000	50	.0075	.000001	.0010	.0144	.0099
182	2000	50	.0075	.000001	.0025	.0237	.0118
183	2000	50	.0075	.000001	.0100	.0522	.0124
184	2000	50	.0075	.000002	.0010	.0144	.0115
185	2000	50	.0075	.000002	.0025	.0237	.0153
186	2000	50	.0075	.000002	.0100	.0522	.0169
187	2000	50	.0075	.000005	.0010	.0144	.0149
188	2000	50	.0075	.000005	.0025	.0237	.0218
189	2000	50	.0075	.000005	.0100	.0522	.0272
190	2000	100	.0000	.000001	.0010	.0105	.0046
191	2000	100	.0000	.000001	.0025	.0175	.0046
192	2000	100	.0000	.000001	.0100	.0398	.0050
193	2000	100	.0000	.000002	.0010	.0105	.0076
194	2000	100	.0000	.000002	.0025	.0175	.0076

DEFINITION OF PARAMETERS

m = SAMPLING FREQUENCY
s = SAMPLE SIZE
a = INHERENT PROBABILITY OF THE PROCESS PRODUCING A DEFECTIVE ITEM
b = RATE AT WHICH THE PROBABILITY OF PRODUCING A DEFECTIVE ITEM INCREASES OVER TIME
cl = DESIRED CENTERLINE VALUE ON CONTROL CHART (ROUGH ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE)
UCL = COMPUTED UPPER CONTROL LIMIT FOR CONTROL CHART
p = RESULTING ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE

Case	m	s	a	b	cl	UCL	p
195	2000	100	.0000	.000002	.0100	.0398	.0097
196	2000	100	.0000	.000005	.0010	.0105	.0117
197	2000	100	.0000	.000005	.0025	.0175	.0117
198	2000	100	.0000	.000005	.0100	.0398	.0192
199	2000	100	.0050	.000001	.0010	.0105	.0086
200	2000	100	.0050	.000001	.0025	.0175	.0086
201	2000	100	.0050	.000001	.0100	.0398	.0099
202	2000	100	.0050	.000002	.0010	.0105	.0109
203	2000	100	.0050	.000002	.0025	.0175	.0109
204	2000	100	.0050	.000002	.0100	.0398	.0143
205	2000	100	.0050	.000005	.0010	.0105	.0145
206	2000	100	.0050	.000005	.0025	.0175	.0145
207	2000	100	.0050	.000005	.0100	.0398	.0222
208	2000	100	.0075	.000001	.0010	.0105	.0106
209	2000	100	.0075	.000001	.0025	.0175	.0106
210	2000	100	.0075	.000001	.0100	.0398	.0123
211	2000	100	.0075	.000002	.0010	.0105	.0125
212	2000	100	.0075	.000002	.0025	.0175	.0125
213	2000	100	.0075	.000002	.0100	.0398	.0165
214	2000	100	.0075	.000005	.0010	.0105	.0159
215	2000	100	.0075	.000005	.0025	.0175	.0159
216	2000	100	.0075	.000005	.0100	.0398	.0237
217	2000	150	.0000	.000001	.0010	.0087	.0042
218	2000	150	.0000	.000001	.0025	.0147	.0048
219	2000	150	.0000	.000001	.0100	.0344	.0050
220	2000	150	.0000	.000002	.0010	.0087	.0063
221	2000	150	.0000	.000002	.0025	.0147	.0082
222	2000	150	.0000	.000002	.0100	.0344	.0099
223	2000	150	.0000	.000005	.0010	.0087	.0093
224	2000	150	.0000	.000005	.0025	.0147	.0121
225	2000	150	.0000	.000005	.0100	.0344	.0201
226	2000	150	.0050	.000001	.0010	.0087	.0078
227	2000	150	.0050	.000001	.0025	.0147	.0090
228	2000	150	.0050	.000001	.0100	.0344	.0100
229	2000	150	.0050	.000002	.0010	.0087	.0092
230	2000	150	.0050	.000002	.0025	.0147	.0114
231	2000	150	.0050	.000002	.0100	.0344	.0147
232	2000	150	.0050	.000005	.0010	.0087	.0120
233	2000	150	.0050	.000005	.0025	.0147	.0149

DEFINITION OF PARAMETERS

m = SAMPLING FREQUENCY
s = SAMPLE SIZE
a = INHERENT PROBABILITY OF THE PROCESS PRODUCING A DEFECTIVE ITEM
b = RATE AT WHICH THE PROBABILITY OF PRODUCING A DEFECTIVE ITEM INCREASES OVER TIME
cl = DESIRED CENTERLINE VALUE ON CONTROL CHART (ROUGH ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE)
UCL = COMPUTED UPPER CONTROL LIMIT FOR CONTROL CHART
p = RESULTING ESTIMATE OF THE PROCESS AVERAGE FRACTION DEFECTIVE

Case	m	s	a	b	cl	UCL	p
234	2000	150	.0050	.000005	.0100	.0344	.0231
235	2000	150	.0075	.000001	.0010	.0087	.0096
236	2000	150	.0075	.000001	.0025	.0147	.0110
237	2000	150	.0075	.000001	.0100	.0344	.0124
238	2000	150	.0075	.000002	.0010	.0087	.0108
239	2000	150	.0075	.000002	.0025	.0147	.0130
240	2000	150	.0075	.000002	.0100	.0344	.0169
241	2000	150	.0075	.000005	.0010	.0087	.0133
242	2000	150	.0075	.000005	.0025	.0147	.0162
243	2000	150	.0075	.000005	.0100	.0344	.0246

LIST OF REFERENCES

1. Duncan, Acheson J., *Quality Control and Industrial Statistics,* Richard D. Irwin, Inc., Homewood, Illinois, 1974.

2. Department of Defense Military Standard MIL-STD 105D, *Sampling Procedures and Tables for Inspection by Attributes,* 1961.

3. Grant, Eugene L. and Leavenworth, Richard S., *Statistical Quality Control* McGraw-Hill, Inc., New York, New York, 1980.

4. Department of Defense Military Standard MIL-STD 414, *Sampling Procedures and Tables for Inspection by Variables for Percent Defective,* 1957.

5. Brugger, Richard M., *"A Simplified Markov Chain Analysis of ANSI ASQC Z1.4 Used Without Limit Numbers", Journal of Quality Technology* , April 1989, pp.97-102.

6. Oakland, John S., *Statistical Process Control,* John Wiley and Sons, Inc., New York, New York, 1986.

7. Ross, Sheldon M., *Introduction to Probability Models,* Academic Press, Inc., San Diego, California, 1985.

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